

الدوال العكسية

Properties of Functions:

- ① $\begin{cases} y = 2x + 5 \rightarrow \text{Relation} \\ x^2 + y^2 = 4 \rightarrow \text{Function} \end{cases}$ (each vertical line intersect with function in one point)
(each value of x have one value of y).

→ Is every function have inverse?

x is function of y (when each value of x has one and it is converse value of y).

- ② $\begin{cases} y = x^2 \text{ (not inv.)} \\ y = 2x + 5 \text{ (inv.)} \end{cases}$ (each horizontal line intersect curve in one point).

→ one to one function. (y is function of x)
(the only function that (x is function of y) has inverse). } The condition one to one

$$\begin{cases} \text{if } x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \\ f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \end{cases} \text{ definition.}$$

⇒ From (1) $2x_1 + 5 = 2x_2 + 5 \Rightarrow x_1 = x_2$ (one to one func)

⇒ From (2) $x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \Rightarrow x_1 \neq x_2$ (not one to one)

$y = x^2$ ($x \geq 0$) \Rightarrow is one to one when there is a condition in specific interval.

→ The relation between function and inverse?

$y = f(x) \rightarrow \text{function.}$

$x = f^{-1}(y) \rightarrow \text{The Form of inverse function.}$

from (1) $f(x) = 2x + 5$ $x = f^{-1}(y) = \frac{y-5}{2}$
 $f^{-1}(x) = \frac{x-5}{2}$

→ The relation between function and inverse? (f and f^{-1})

- Domain of f is range of f^{-1}
- Range of f is domain of f^{-1} .
- Graph is symmetric about ($y=x$)

$$f^{-1}(f(x)) = x, \quad f(f^{-1}(x)) = x$$

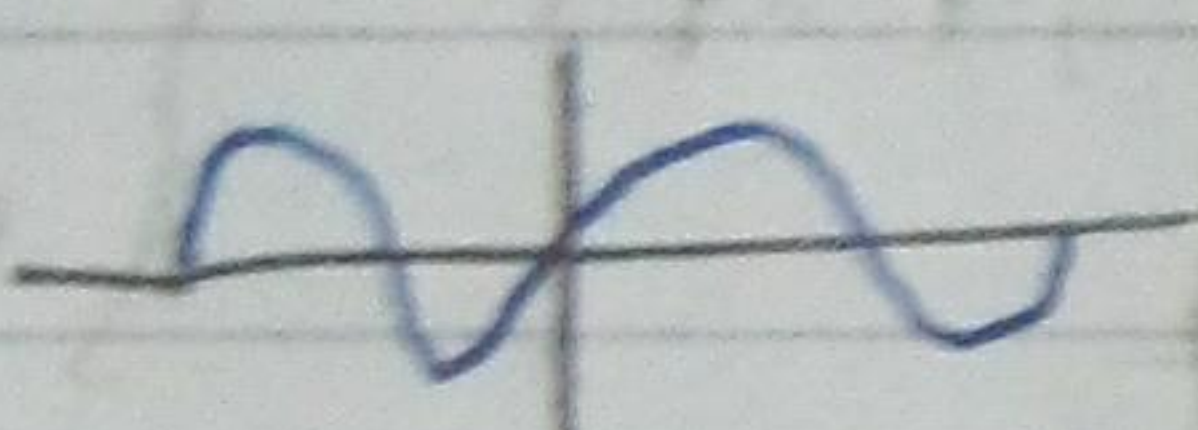
- Inverse function: is a function of (one to one function)
→ if a function isn't one to one (There is an inverse function of specific interval).

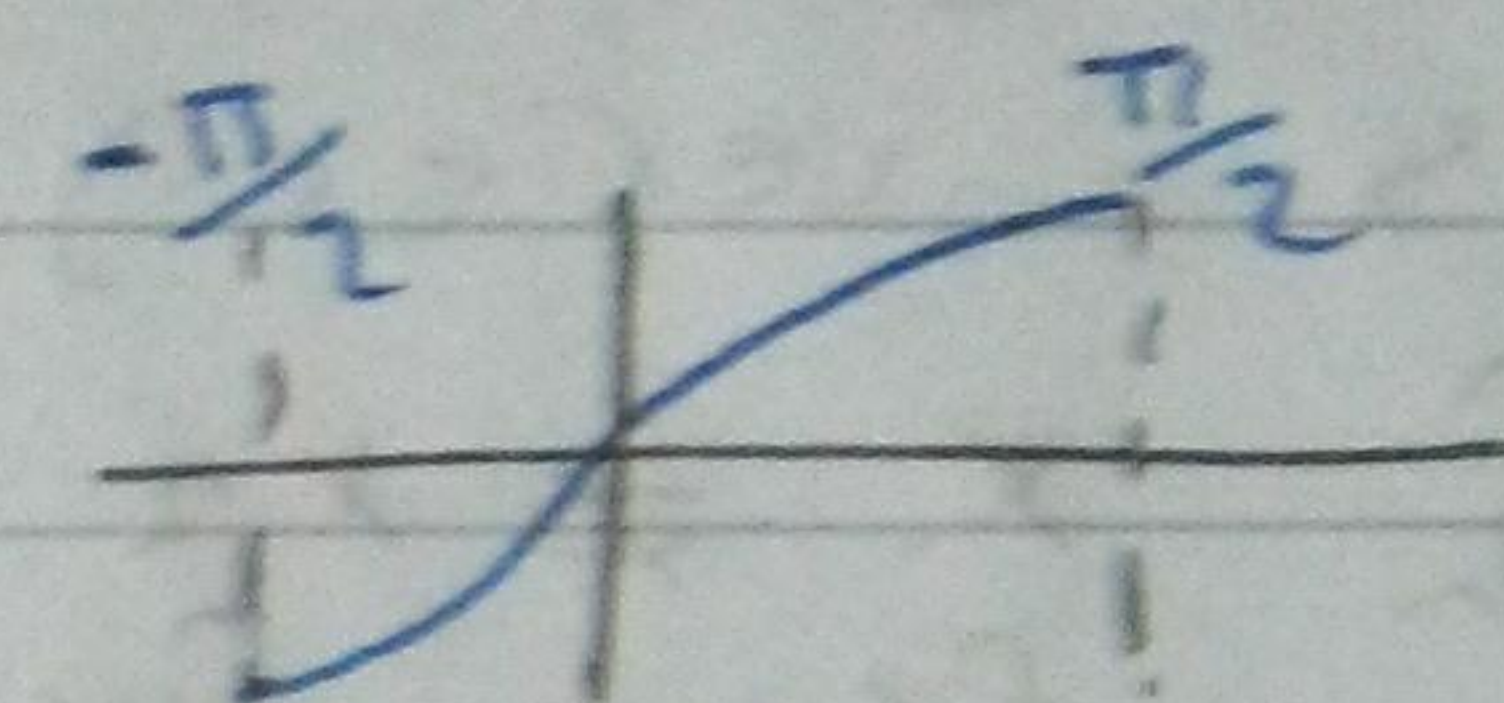
- Inversed Trigonometric functions:

$y = \sin x$ $\xrightarrow{\text{inverse}}$ $y = \sin^{-1} x \neq \frac{1}{\sin x} = (\sin x)^{-1}$

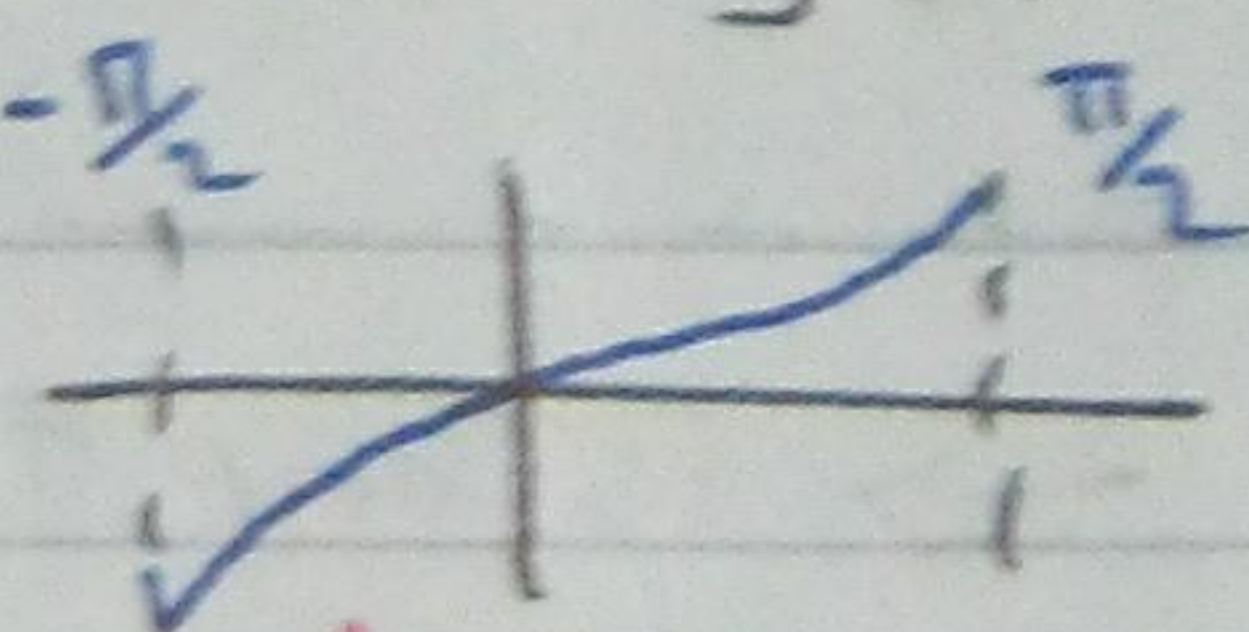
Domain: \mathbb{R} $\xrightarrow{\text{restriction}}$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to be one to one function.

Range: $[-1, 1]$

Graph: 
(not one to one)

Graph: 

③ $y = \sin^{-1} x \Rightarrow D: [-1, 1], \text{ Range } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

→ Graph: 

→ Derivative: $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

⇒ From ③

$$\sin y = \sin(\sin^{-1} x) = x$$

$$x = \sin y$$

$$\frac{dx}{dy} = \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Derivative: $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

Note
 $\frac{dy}{dx} = \left(\frac{1}{\frac{dx}{dy}} \right)$

$$y = 2x + 1$$

$$\frac{dy}{dx} = 2$$

$$x = \frac{y-1}{2}$$

$$\frac{dx}{dy} = \frac{1}{2}$$

(complete derivative)

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \text{if } \frac{d}{dx} (\sin^{-1} u(x)) = \frac{u'(x)}{\sqrt{1-u^2(x)}}$$

→ $u'(x)$ → Derivative of function but (u not u^2).

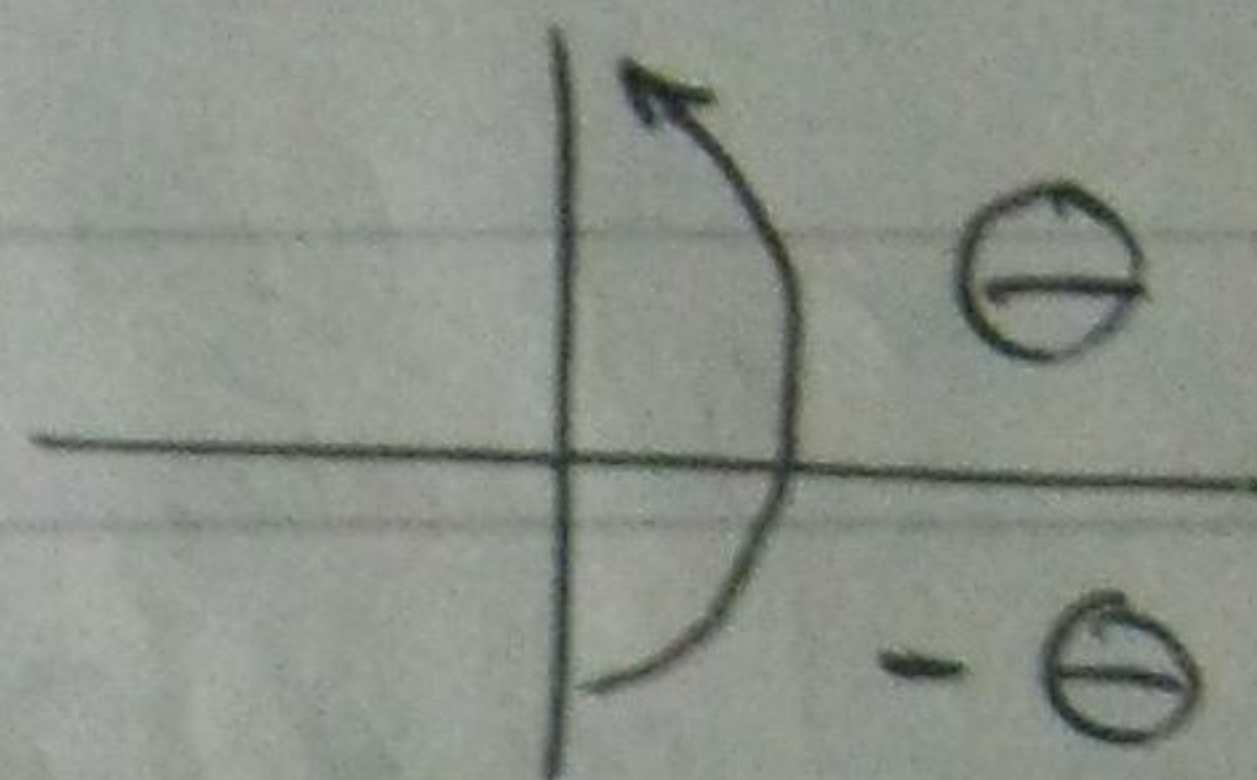
$$\sin x = \frac{1}{2} \quad \begin{array}{|c|} \hline 1 \\ \hline \sqrt{3} \\ \hline \end{array}$$

$$\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

as Range of $\sin^{-1} x$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$

$$\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$$

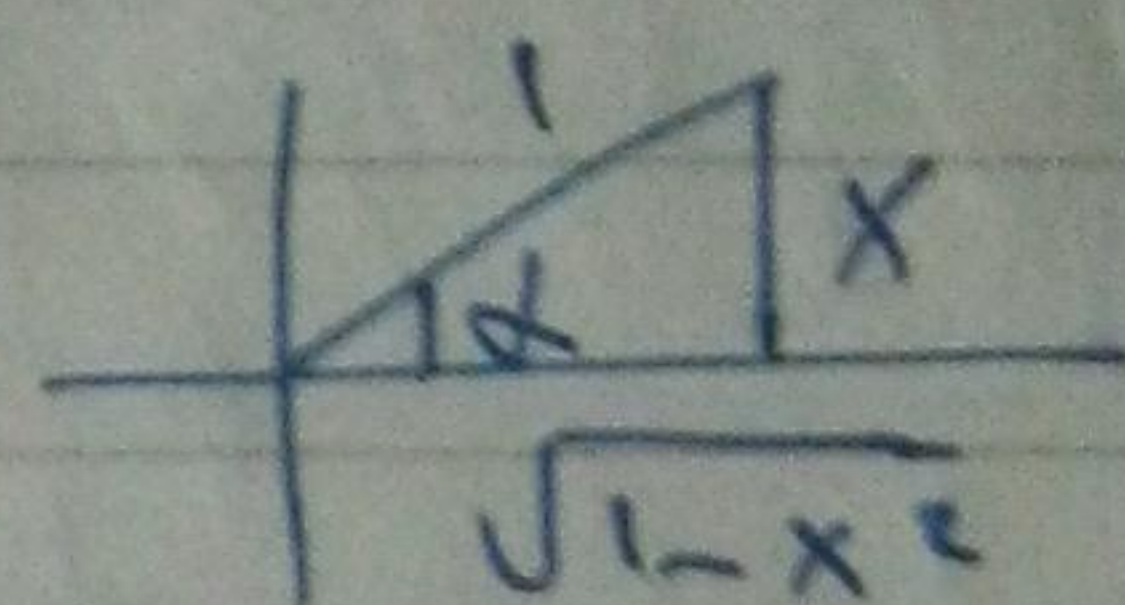
Note: each angle in the fourth quarter is (-ve).



as Domain Range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. not $[0, 2\pi]$.

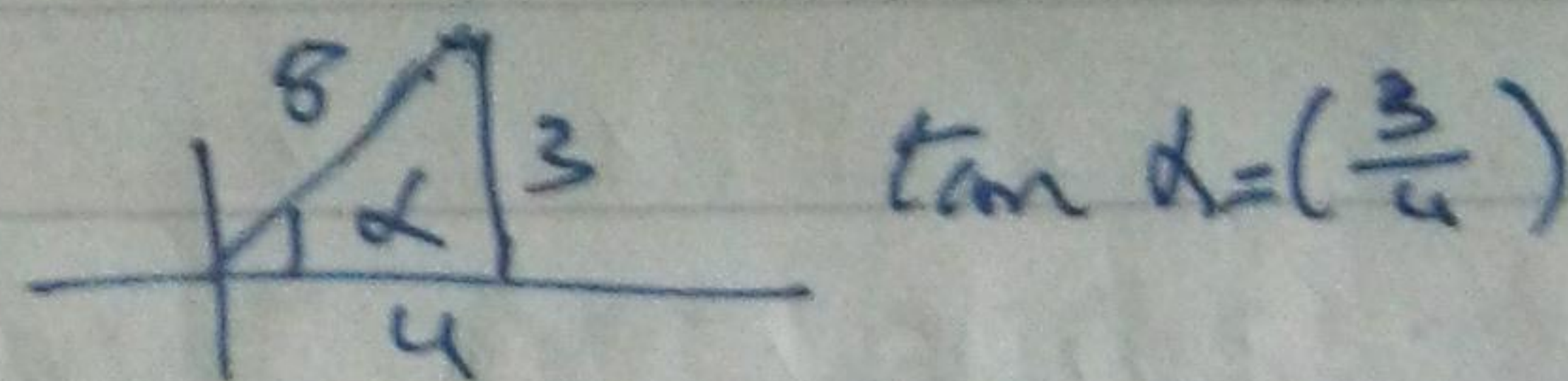
$$\cos = (\sin^{-1} x) \Rightarrow \sin^{-1} x = \alpha$$

$$x = \sin \alpha.$$

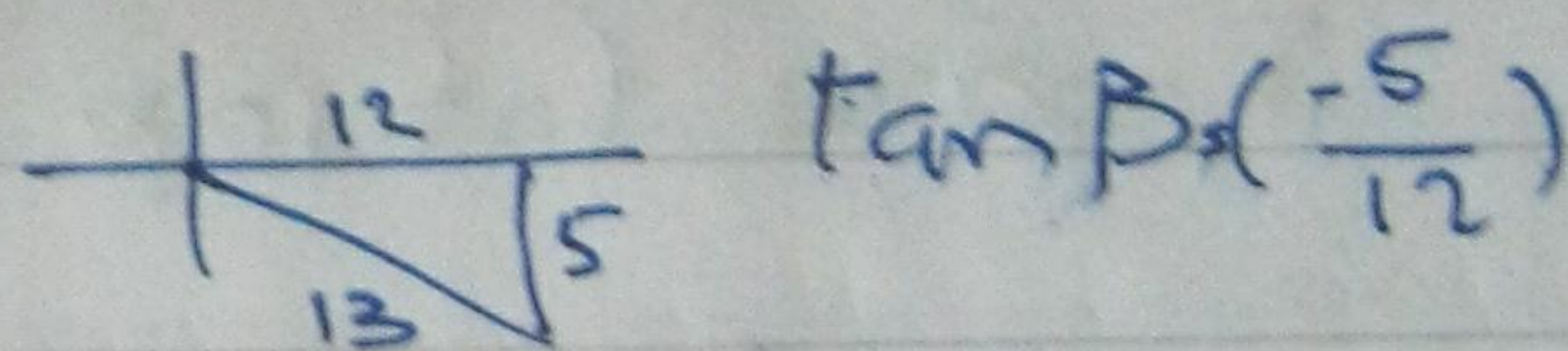


ex $\Rightarrow \tan(\sin^{-1} \frac{3}{5} + \sin^{-1}(-\frac{5}{13}))$

1) $\sin^{-1} \frac{3}{5} = \alpha$ $\sin \alpha = \frac{3}{5}$



2) $\sin^{-1}(-\frac{5}{13}) = \beta$ $\sin \beta = -\frac{5}{13}$



$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{(\frac{3}{4}) + (-\frac{5}{12})}{1 - (\frac{3}{4})(-\frac{5}{12})}$$

ex $\Rightarrow y = \sin^{-1}(x^2)$

$$y' = \frac{2x}{\sqrt{1-(x^2)^2}}$$

$(x^2)^2 \Rightarrow$ The function squared

$(2x) \Rightarrow$ derivative of function.
not function's squared

ex $\Rightarrow y = \sin^{-1}(e^{2x})$

$$y' = \frac{2e^{2x}}{\sqrt{1-(e^{2x})^2}}$$

ex $\Rightarrow y = \sin^{-1}(\sqrt{x^2+1})$

$$y' = \frac{\frac{2x}{2\sqrt{x^2+1}}}{\sqrt{1-(x^2+1)}}$$

$$\underline{\text{ex}} \Rightarrow y = \sin^{-1}(\cos x).$$

$$y' = \frac{-\sin x}{\sqrt{1 - \cos^2 x}} =$$

$$= \frac{-\sin x}{\sin x} = \underline{\underline{-1}}$$